Optimal toeholds in takeover contests

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Abstract

We offer an explanation for why raiders do not acquire the maximum possible toehold prior to announcing a takeover bid. By endogenously modeling the target firm’s value following an unsuccessful takeover we demonstrate that a raider may optimally acquire a small toehold even if the acquisition does not drive up the pre-tender target price. This occurs because although a larger toehold increases profits if the takeover succeeds it also conveys a higher level of managerial entrenchment and hence a lower firm value if the takeover fails. We derive new predictions regarding the optimal toehold and target value following a failed takeover. We also examine the impact of a rival bidder and dilution.

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1. Introduction

For the past several decades, takeover contests have attracted the interest of academics, practitioners, and regulators. This has led to an impressive array of both theoretical and empirical research that has greatly enhanced our understanding of the basic mechanisms behind the takeover process; Hirshleifer (1995) and Shleifer and Vishny (1997) offer surveys of this literature. However, some questions remain unanswered. In particular, there seems to be a gap between theory and empirical studies when it comes to the issue of toeholds. A toehold is defined as the raider’s ownership stake in the target firm prior to announcing his tender offer. SEC regulation specifies that anyone who acquires 5% of a company’s outstanding shares must file Schedule 13(D) within ten days to disclose their identity, the number of shares owned, and their purpose in acquiring the shares. In practice, a bidder can continue purchasing target shares anonymously after hitting the 5% threshold until the disclosure date.

While classic theory predicts that raiders should accumulate the maximum legally allowed toehold prior to making a public takeover bid, empirically we see that toehold size varies widely. For example, Bradley et al. (1988), Stulz et al. (1990), Jennings and Mazzeo (1993), and Betton and Eckbo (2000) all report that a significant fraction of raiders hold toeholds well below 5% when the takeover is announced, with some bidders having no toeholds at all. This is surprising since Jennings and Mazzeo (1993), Schwert (1996), and Betton and Eckbo (2000) all show that raiders pay a high premium over market value for target shares during the tender offer process.

In this paper we provide a theoretical explanation for this empirical anomaly. We accomplish this by expanding on previous work in two ways. First, we assume that the manager of the target firm can strategically decide whether to oppose the tender offer and reduce the likelihood of takeover success. Second, we explicitly model the value of the target firm following a failed takeover.¹ These two features allow us to demonstrate that while a larger toehold increases the raider’s profits if the takeover succeeds, it can also reduce his profits if the takeover fails. This previously unmodeled cost of a larger toehold can result in an optimal toehold that is less than the 5% legal threshold. Furthermore, in some cases the raider will optimally bid for the target while acquiring a zero toehold.

Thus, we rationalize the existence of an interior solution to the optimal toehold problem as well as generate a new set of empirical predictions regarding the cross-sectional variation in toehold size and in firm value following a failed takeover. In addition, we provide testable implications regarding changes in the price of the target firm for different event windows during the takeover process.

The intuition for our findings is based on the observation that the target manager’s incentive to block a takeover increases with the level of her private benefits of

¹Grossman and Hart (1980), Shleifer and Vishny (1986), Hirshleifer and Titman (1990), and Chowdhry and Jegadeesh (1994) all assume that firm value after a failed takeover is the same as that before the takeover is announced.
control. Following a failed takeover, then, the market will reevaluate its assessment of the level of managerial entrenchment (private benefits) and therefore the expected future value of the firm. Because ex ante a takeover is more likely to succeed when the raider has a larger toehold, a failed takeover in which the raider had acquired a larger toehold will convey to the market, ex post, that the level of entrenchment must be high as well. Since high levels of entrenchment reduce firm value, the larger the toehold, the lower will be the market’s assessment of the firm’s value following a failed takeover. Therefore, the raider will increase his toehold to the point where the marginal benefit from buying an additional share when the takeover succeeds is equal to the marginal cost when the takeover fails. Implicit in this argument is that unlike shares tendered at the takeover stage, the raider owns the toehold whether the takeover is successful or not.

Our model relies on two key assumptions. First, we assume that higher levels of managerial entrenchment imply a lower firm value to outside shareholders. Although some papers have argued that managerial entrenchment can actually increase firm value, the empirical evidence on termination of takeovers by management resistance and on adoption of anti-takeover measures seems to suggest otherwise. Mikkelson and Ruback (1985), Ruback (1988), Choi (1991), and Safieddine and Titman (1999) show that stock prices respond negatively to an announcement of the failure of a takeover attempt. In particular, Safieddine and Titman find that less than 50% of firms that reject an initial takeover are taken over within the next five years. Thus, they conclude that the expected marginal gain from a potentially higher tender offer in the future cannot compensate for the loss from rejecting the initial offer.

Taking a different approach, Denis and Serrano (1996) find that of the firms that experience a failed takeover, those in which target managers remain in control (66%) are characterized by ineffective block-shareholder monitoring and by subsequent underperformance relative to firms that replace their managers. They conclude that this is evidence of value-reducing entrenchment.

Looking at anti-takeover measures, Ryngaert (1988) and Malatesta and Walkling (1988) both find a negative stock price reaction to the adoption of poison pills. Linn and McConnell (1983) find that the adoption of supermajority rules and staggered boards actually leads to a positive revaluation of stock prices, while Jarrell and Poulson (1987) find that the adoption of fair price amendments (non-fair price amendments including supermajority rules) induce insignificant (negative) reaction to stock prices. Pound (1987) finds that the adoption of poison pills deters takeovers, while Comment and Schwert (1995) find no evidence of a reduction in the probability of takeover success. Finally, Jennings and Mazzeo (1993) show that, although management resistance increases the likelihood of the arrival of competing bids, this increased probability does not justify the expected wealth loss from rejecting bids that are already on the table.

Our second key assumption is that the probability of takeover success increases with the size of the toehold. This is consistent with the theoretical prediction of Hirshleifer and Titman (1990), as well as with the empirical evidence found in Walkling (1985), Choi (1991), Jennings and Mazzeo (1993), and Betton and Eckbo (2000).
The model generates two main sets of testable implications. First, since we solve for the optimal toehold we are able to demonstrate how the size of the toehold should vary cross-sectionally. For example, we predict that the optimal toehold decreases with the target management’s equity ownership (i.e., insider ownership). The intuition for this is that higher ownership by target managers increases their incentive to support a value-increasing takeover.\(^2\) Therefore, when the market observes a failed takeover of a target firm with high managerial ownership it concludes that the level of entrenchment must be very high as well. Hence, the market’s revised assessment of target value decreases with target management’s ownership. Since this value represents the price at which the raider can sell his toehold, the lower is this value the lower will be the expected profit from buying the toehold. Thus, there is a negative relation between toehold size and insider ownership of the target. An additional result is that the size of the toehold should be larger when the raider is expected to have more private benefits from controlling the target firm. To our knowledge, these as well as our other predictions about toehold size have not yet been tested.

Our second set of predictions concerns the value of the target firm following a failed takeover. For example, our model predicts that the value of the target firm following a failed takeover should be lower for a target firm that has a CEO with a larger equity stake and for a firm with a lower expected level of agency problems. This is because both larger insider ownership and a lower expected level of agency problems in the target increase the market’s ex ante assessment of the probability of takeover success. Hence, after observing a failed takeover of a firm with, for example, few expected agency problems the market will reassess the level of managerial entrenchment upward and target value downward. This will imply that we would expect firms with small cash holdings, high levels of debt, and strong corporate governance (all of which proxy for less severe agency problems) to exhibit the largest price declines following the announcement of a failed takeover. While, again, many of these predictions have not yet been tested, previous empirical work such as Safieddine and Titman (1999) has shown that firm value decreases following a failed takeover. In addition, Safieddine and Titman also show that firm performance (which in our model represents firm value) decreases after a failed takeover and that this decrease in performance is larger for firms with higher levels of insider ownership.

\(^2\)We assume that higher target ownership implies that the manager’s interests are more aligned with those of minority shareholders and hence that the ex ante probability of takeover success increases with the target managers equity stake (see Assumption 5). Empirical evidence (e.g., Morck et al., 1988; Servaes and McConnell, 1990) and theoretical work (e.g., Stulz, 1988) suggest that for some range of ownership there is a negative relation between ownership and firm value. Given the evidence in Mikkelson and Partch (1989), who show that higher insider ownership leads to a higher rate of takeover success, we focus in our model on the positive relation between target insider ownership and firm value when deriving comparative static results. If, however, one believes that ownership is in the range where higher ownership leads to higher entrenchment, then our comparative static results regarding the impact of a change in target ownership will go the other way.
Finally, our model assumptions generate several interesting implications regarding the bid premium and the magnitude of target price changes for different event windows surrounding the takeover contest. For example, we find that the takeover premium (the price difference between the offer price and the pre-announcement price) should be lower when the raider’s expected private benefits are high (and hence toehold size is high too), but that the price jump at the announcement of the takeover should be higher for targets with more severe agency problems. (These predictions can be generated from existing takeover models, but we present them alongside our other new predictions for the sake of completeness.) While most of these implications have not yet been tested, Betton and Eckbo (2000) find that a higher premium is associated with smaller toeholds. This finding is consistent with our model as well as with the models of Shleifer and Vishny (1986) and Hirshleifer and Titman (1990).

In two extensions of the basic model we study how the results change when the raider is able to dilute minority target shareholders and when we consider the possible presence of a rival bidder. In the case of dilution we show that, not surprisingly, the raider is able to win the takeover by offering a lower bid and hence is able to generate higher profits (e.g., Grossman and Hart, 1980). In the case of multiple bidders we show that the initial bidder will acquire a larger toehold when the presence of a rival bidder with a higher valuation is expected. This is because the initial bidder can now also profit from selling his toehold to the rival bidder.

The rest of the paper is organized as follows. In the next section we review the literature on takeovers and toeholds. We then present the general takeover model in Section 3, and in Section 4 we solve a specific takeover model and describe the resulting empirical predictions. In Section 5 we extend the model to include dilution and a rival bidder. Section 6 concludes. All proofs are left to the Appendix.

2. Literature review

There is an extensive literature on takeovers and corporate control. Grossman and Hart (1980) show how the free-rider problem can prevent any value-enhancing takeovers from taking place. They conclude that the free-rider problem can be overcome by including a corporate charter that allows raiders to dilute minority shareholders. Shleifer and Vishny (1986) and Hirshleifer and Titman (1990) demonstrate that acquiring a toehold in the target prior to making the takeover bid can solve the free-rider problem. These papers assume that the raider’s toehold is exogenously given (else they would get a corner solution). We add to this literature by identifying a cost to acquiring a large toehold: While the toehold remains the source of profits for the raider, it also affects the magnitude of the stock price decline following a failed takeover.

Two papers that have theoretically addressed the small-toehold anomaly include Chowdhry and Jegadeesh (1994) and Ravid and Spiegel (1999). Chowdhry and Jegadeesh show that when bidder type is private information, a low-valuation bidder might wish to acquire a smaller toehold in order to separate himself from
high-valuation bidders and thus offer a lower premium in the tender offer. Their model predicts a positive relation between the toehold size and the takeover premium, which is inconsistent with Betton and Eckbo (2000) who find that, similar to the prediction of our model, the toehold size should be negatively related to the premium. Additionally, while Chowdhry and Jegadeesh predict that a larger toehold should increase the likelihood of takeover success only through an increase in the bid premium, we argue that the likelihood of takeover success should also be increasing in the size of the toehold. This again is consistent with, for example, Walkling (1985) and Betton and Eckbo (2000) who show that even after controlling for the size of the bid premium, takeover success is increasing in the size of the toehold.

Ravid and Spiegel (1999) study the effects of legal requirements on the second-tier, cleanup price during the merger period. They show that a cost to a toehold is that purchasing shares in the pre-tender market can drive up the pre-tender price and hence increase the price paid following a takeover for all shares that were not tendered (i.e., the cleanup price). In contrast with their model, we show that an interior optimal toehold can exist even with no target price runup prior to the tender offer. In our paper it is the negative marginal impact on firm value following a failed takeover that stops the raider from increasing the toehold size. Empirically, Betton and Eckbo (2000) find that larger toeholds are associated with lower pre-bid target price runups, which is not consistent with Ravid and Spiegel.

Finally, there are two empirically motivated potential explanations for the toehold anomaly. First, Schwert (1996) shows that the pre-bid runup in the target’s stock price is uncorrelated with the post-announcement markup, defined to be the change in target price after the first bid is announced. This evidence suggests that it is costly to buy a large toehold since increasing the size of the toehold will push up the pre-bid price and, given a “fixed” markup, will also increase the offer price. The validity of this argument depends, however, on whether the price runup is indeed caused by the raider’s toehold acquisition. Schwert argues that his results imply that the information revealed during the pre-bid price runup is not information about the acquisition of the toehold, but rather additional information about the value of the target firm that is new to both the raider and the target shareholders (for example, a purchase by another potential raider). In other words, since the pre-bid runup in target price does not lead to a reduction in the post-bid markup (on average), Schwert concludes that the pre-bid price runup also leads the raider to increase his valuation of the target. If Schwert’s conclusion is correct then within the context of our model adding a price runup that is due to the arrival of new information should not have any effect on the decision of the current raider to acquire a toehold. For this reason, while we could easily incorporate this feature into our model and generate the above correlation structure, we choose to leave it out. It remains interesting to see, however, whether the zero correlation found in Schwert holds for both takeovers in which the raider has acquired a large toehold and those in which the raider has not.³

³Interestingly, Betton and Eckbo (2000) and Bris (2001) find a negative relation between target price runup and the size of toehold but they do not test for the correlation with the markup.
A second reason why a raider may choose to avoid buying a toehold is that a toehold acquisition can create hostility with the incumbent target manager. While it is debatable whether hostility is harmful for raiders in acquisitions, if a raider believes that a friendly takeover or merger is indeed a better alternative than a hostile bid then he will forgo toehold acquisition and pursue a friendly offer. An implication of this argument is that bidders in friendly takeovers should have a zero toehold and bidders in hostile bids should have large toeholds. While the empirical evidence on toeholds in hostile takeovers reveals that many bidders still do not have large toeholds, recent empirical research illustrates that in friendly mergers some bidders do acquire toeholds. Thus, the difference between friendly and hostile acquisitions cannot fully explain the empirical findings on toeholds, especially for hostile bids that we focus on.

To summarize, given that recent empirical evidence casts some doubt on whether existing models can explain the toehold anomaly, we argue that our paper offers both an alternative (and perhaps complementary) explanation for the toehold anomaly as well as providing new testable empirical predictions regarding toehold size and firm value following a failed takeover. While Chowdhry and Jegadeesh (1994) and Ravid and Spiegel (1999) show how buying a larger toehold can increase the price at which the raider has to buy his shares at the tendering stage, our new predictions come from demonstrating how a toehold can adversely affect the value of the raider’s position following a failed takeover when shareholders update their belief about the level of managerial entrenchment.

3. The takeover model

We study an all-equity target firm managed by a risk-neutral manager. The manager owns a fraction $\beta$ of the firm’s stock. The remainder of the firm is held by a group of small, dispersed, and risk-neutral shareholders. The target manager derives utility from monetary compensation as well as from private benefits of control. The private benefits, denoted by $C$, can be thought of as a characteristic of the manager that also determines her ability to work for shareholders. In other words, a higher level of $C$ implies both that the manager receives more private benefits and that the value of the firm is lower. For example, a higher level of $C$ can result in higher levels of entrenchment and hence the manager will care less about firm value and more about protecting her job. While $C$ is a constant we assume that it is privately known to the manager but not to the raider or to outside investors. The market initially has a prior belief about the distribution of $C$.

\footnote{For hostile deals, Bradley et al. (1988) find that 66% of the bidders in their sample of 236 successful tender offers have zero toeholds, while Betton and Eckbo (2000) find that 47% of initial bidders in their sample of over 1,300 tender offers (including failed ones) have zero toeholds. For friendly deals (stock mergers), Burch (2001), Bates and Lemmon (2003), and Officer (2003) all report that more than 80% of acquirers have zero toeholds. The higher frequency of zero toeholds in friendly deals is consistent with the above argument of toehold and hostility.}
The firm operates for four periods. At $t = 0$ a raider with private benefits of $C_R$, resulting from a successful takeover, observes a value improvement for the firm, $Z$, which he can implement if he gains control via a hostile takeover. After observing his value improvement, the raider buys a toehold, $\alpha$. At $t = 1$ the raider makes a take-it-or-leave-it bid for a fraction $\omega - \alpha$ of the firm’s shares, where $\omega$ is the fraction of shares needed to gain control.\footnote{While the target manager owns a fraction $\beta$ of the firm’s shares, a takeover requires that the raider obtain the fraction $\omega$ of shares. Although the manager has discretion over decisions such as the launch of a poison pill, this does not come from her ownership $\beta$ but rather from her position as the manager. Therefore, a block transaction in which the raider approaches the manager for her shares alone will not result in a transfer of ownership and control of the target’s assets. In other words, we assume that $\beta \ll \omega$.} If the takeover succeeds (with probability $\phi$) the raider implements his value improvement and the game ends. If the takeover fails (with probability $1 - \phi$) the incumbent manager stays in control, consumes her private benefits, and invests in a project that yields a cash flow to shareholders at $t = 3$. At $t = 2$, the manager leaves the firm and sells her shares at the prevailing market price. Finally, at $t = 3$, the cash flows from the project are realized and the firm is liquidated. Note that we assume that the manager leaves the firm before the final cash flows are realized to capture the notion that the manager cares about the stock price in addition to her private valuation of the firm. Fig. 1 summarizes the sequence of events.

### 3.1. The market for corporate control

The ex ante probability of takeover success, $\phi$, is less than one because the manager can choose to fight the takeover attempt (say by launching a poison pill) if the value increase to her shares is lower than the private benefits of control she would lose. Since the level of these private benefits is known only to her, the raider faces uncertainty regarding the success of the takeover.

To increase the likelihood of success, the raider can strategically choose both the optimal toehold size, $\alpha$, prior to making the tender offer, and the optimal bid, $V_1 + X$, to offer target shareholders for the fraction $\omega - \alpha$ of the firm’s stock. The value $V_1$ reflects the value of the firm if the takeover fails and $X$ is the spread above this value that the raider pays in order to convince the manager (and shareholders) to...
tender. Since the manager compares $V_1$ to $V_1 + X$ when deciding on whether to support the takeover, we define the value improvement $Z$ over $V_1$ rather than over the initial stock price. We assume that the tender offer is restricted and conditional. This means that the raider accepts up to a fraction $\omega$ of the firm’s stock only if he acquires enough shares to control the firm (if shareholders tender more than $\omega$ shares they will be purchased on a pro rata basis). Otherwise he is left with the toehold purchased before the tender offer. We further assume that the raider can purchase the toehold anonymously at $V_0$ as long as the toehold size $\bar{a}$ does not exceed the maximum level, $\bar{a}$, that he is allowed to acquire by law without having to disclose his intentions.

At $t = 1$ the raider’s valuation, $V_1 + Z$, is private information. However, we assume that this information, along with the toehold size, becomes public knowledge once the tender offer is announced. This implies that the raider can accumulate a toehold at a price of $V_0$ prior to making the bid but that he cannot do so once he announces the tender offer. The assumption that $Z$ becomes public information at the time of the tender offer is for technical convenience but can also be justified based on the legal environment regarding information disclosure surrounding tender offers in the U.S.\(^6\)

Minority target shareholders are non-pivotal, and hence a necessary condition for a takeover to succeed is that the raider offers shareholders a price of $V_1 + X$ that satisfies $V_1 + X \geq V_1 + Z$ (Grossman and Hart, 1980). This condition, however, is not sufficient for a successful takeover because the target manager can strategically launch defense measures and block the takeover. She does so if she is better off staying on as CEO of the firm. In general, the target manager faces a tradeoff between supporting the takeover and getting the premium on her equity stake (but being replaced by the raider) and opposing the takeover and getting her control benefits.\(^7\) This implies that the more equity the manager has the less likely she is to oppose the takeover. Finally, we assume that any attempt by the manager to block a takeover will become more difficult the larger is the equity stake of the raider.

Empirically, it has been shown that the ownership distribution of target shares is an important factor in determining the outcome of a takeover contest; Jensen and Warner (1988) provide an excellent survey. In particular, Mikkelson and Partch (1989) find that higher ownership by the target manager increases the probability of a successful takeover, Jennings and Mazzeo (1993) find that toeholds reduce the likelihood of resistance by management, and Betton and Eckbo (2000) find that a higher takeover premium and a larger toehold increase the probability of a successful takeover.

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\(^6\)The SEC rules and regulations require that when a raider announces a tender offer he must file Schedule 14(D) on the same day and disclose his plans for the target firm following the takeover. Moreover, the tender offer has to stand for 20 business days so that target shareholders have time to digest all the information and rival bidders have time to enter the contest.

\(^7\)Martin and McConnell (1991) and Denis and Denis (1995) show that successful takeovers result in the replacement of the targets’ CEOs. The target manager can also be replaced following a failed takeover but at a significantly lower probability (e.g., Agrawal and Walkling, 1994; Denis and Denis, 1995; Denis and Serrano, 1996; Safieddine and Titman, 1999).
Betton and Eckbo employ a multinomial logit model (Table 8 of their paper) and find that when a bidder has a positive toehold, the probability of success increases by 15.6% as compared to no toehold at all, while the probability of success increases by 0.012% as the size of the toehold increases by one percentage point from the mean (e.g., from 5% to 6%). In addition, they find that when the initial offer premium increases by one percentage point from the mean (e.g., from 30% to 31%), the probability of success increases by 0.11%. Finally, Mikkelson and Partch (1989) find that target managers’ ownership is (weakly) positively related to the likelihood that the initial bidder will be successful. In terms of implied probabilities they find that the average probability of a successful offer increases monotonically from 0.77 when managers and directors control less than 10% of votes to 0.91 when they control more than 50% of votes. In light of this empirical evidence we make the following assumption about the functional form of the takeover probability.

**Assumption 1.** The probability of a successful takeover, \( \phi \), is given by \( \phi = \text{Prob}[C < f(\beta, \alpha, X), V_1 + X \geq V_1 + Z] = \phi(\beta, \alpha, X) \) and is increasing in all three variables.

The definition of the probability is for any value of \( \alpha \) and \( X \). In equilibrium, of course, the size of the toehold as well as \( X \) will depend on other exogenous parameters. The key feature of Assumption 1 is that a manager with higher private benefits of control is more likely to actively oppose and block a takeover attempt. Furthermore, the incentive to block a takeover will decrease when the spread, \( V_1 + X - V_1 = X \) is higher and when the manager owns a larger fraction of the firm’s equity. Finally, the larger the toehold the less likely is it that the target manager will defeat the raider in the control contest.

Interestingly, the only uncertainty from the perspective of both the raider and the target shareholders regarding the likelihood of a successful takeover is related to the unknown level of managerial entrenchment, \( C \). In other words, the uncertainty faced by the market arises from the incentive of the target manager to *use* a poison pill (for example) rather than the fact that she *has* a poison pill. In summary, a takeover will succeed if and only if the target shareholders view the tender price as fair *and* the target manager agrees to the takeover.

Assumption 1 above leads to a relation between the perceived level of managerial private benefits following a failed takeover and the size of the toehold (as well as \( \beta \) and \( X \)). Everything else equal, a larger toehold implies a higher ex ante likelihood of a successful takeover. Thus, a failed takeover in which the raider has a large toehold must imply that the manager’s private benefits are also high. In particular, a failed takeover implies that \( C > f(\beta, \alpha, X) \) and since \( f(\cdot) \) is increasing in \( \alpha \) the argument follows. After a failed takeover the market’s expectation is that \( E[C|C > f(\cdot, X)] = C^*(\cdot, \alpha) \) which is increasing in \( \alpha \). Here \( C^* \) denotes the sufficient statistic that

\[^8\text{Jennings and Mazzeo (1993) find that larger toeholds and higher bid premiums reduce the likelihood of resistance by target management, but they do not provide enough information for us to calculate the marginal impact of these variables on the likelihood of target resistance.}\]
summarizes all the value-relevant information about the *perceived* level of private benefits.

It is important to note that one of the important features of our model is that we allow for the manager to affect the probability of takeover success. This implies that the raider must offer a takeover premium that is high enough to induce the manager not to oppose the takeover attempt. This however does not imply that the raider can take control of the firm by buying out the shares of the manager, $\beta$, as is the case in Berkovitch and Khanna (1991), Sercu and Van Hulle (1995), and Dewatripont (1993).

### 3.2. Firm value

The value of the firm at $t = 0$ is $V_0$ while firm value after a failed takeover, at $t = 1$, is denoted by $V_1$ (if the takeover succeeds the value of the firm is $V_1 + X$). Unlike previous models we derive the value $V_1$ endogenously based on the market’s assessment of the expected cash flows from the subsequent investment made by the incumbent manager. Specifically, we let $V_1 = V_1(C^*)$.

**Assumption 2.** Firm value following a failed takeover, $V_1(C^*)$, is a decreasing function of $C^*$.

Assumption 2 reflects our departure from previous work. Past models assume that $V_1(C^*)$ is a constant parameter that is independent of any decision variables. In contrast, we assume that this value depends on the updated market belief about the level of managerial private benefits of control. In particular, from Assumption 1 we know that a failed takeover implies that $C > f(\beta, \alpha, X)$. Thus, Assumption 2 asserts that the higher the target manager’s perceived private benefits, the lower is the value of the firm. This will be true if, for example, higher private benefits imply that future profitable takeovers are less likely to succeed (entrenchment) or if higher private benefits are the result of a manager who spends less time in finding value-improving investments.

Assumptions 1 and 2 highlight the two ways in which the target manager’s private benefits of control affect firm value. First, large private benefits reduce the likelihood of a successful takeover. Second, a high perceived level of private benefits reduces shareholders’ expected value of future cash flows generated by the incumbent manager.

### 3.3. The raider’s optimization problem

After observing his value improvement, the raider chooses the size of the toehold and the tender offer premium to maximize the following objective function:

$$\max_{\{\alpha, X\}} \Pi = \phi[\omega(V_1 + Z) - (\omega - \alpha)(V_1 + X) + C_R] + (1 - \phi)\alpha V_1 - \alpha V_0. \quad (1)$$

Eq. (1) is the raider’s expected profit from a takeover. The last term denotes the cost of acquiring shares at the initial price $V_0$. The first term is the profit from having
a toehold and bidding for $\omega - \alpha$ shares at a price of $V_1 + X$. The second term is the profit from having the toehold after the takeover fails. Eq. (1) can be simplified to
\[
\max_{[\omega, X]} \phi \omega (Z - X) + \alpha \phi X + \alpha (V_1 - V_0) + \phi C_R.
\] (2)

### 3.4. Equilibrium

Target shareholders, upon seeing $\omega$, $Z$, and $V_1 + X$, have to decide whether to tender their shares, while the manager, knowing $C$ and seeing $\omega$, $Z$, and $V_1 + X$, has to decide whether to oppose the takeover. Given rational expectations of the raider and shareholders regarding the function $V_1 = V_1[C^*(\omega, \beta, X)]$ we can derive the raider’s optimal bidding strategy. Since shareholders are non-pivotal and since $Z$ is public information at the time of the tender offer it is clear that shareholders will tender their shares only if $V_1 + X > V_1 + Z$. Recalling that $V_1$ is a function of $C^*$ and that $C^*$ is a function of $X$ and taking the first-order condition in (2) with respect to $X$, we have
\[
\frac{d\Pi}{dX} = \phi X \omega (Z - X) + (\alpha - \omega) \phi + \alpha X \phi + \alpha V_1 X + \phi C_R.
\] (3)

A sufficient condition for which the raider finds it optimal to offer $X^* = Z$ is that
\[
\frac{d\Pi}{dX} \bigg|_{X=Z} = (\alpha - \omega) \phi + (\alpha Z + C_R) \phi + \alpha V_1 X < 0.
\] (4)

The argument for potentially choosing to bid above $V_1 + Z$ is that a higher bid increases the chance that the takeover will succeed and that the raider will be able to profit on his toehold and private benefits while losing on the shares acquired at the tendering stage. Eq. (4) is a sufficient condition for this not to be the case. Since the only positive term is the middle one Eq. (4) states that as long as $\phi_X$ is small the raider’s optimal bid will not exceed his valuation of the firm. For simplicity, we assume this condition holds so that $X^* = Z$. Later, when we solve for a specific formulation of the model, we will see that this condition is satisfied under a very natural assumption. The optimal toehold decision of the raider can now be derived by solving
\[
\max_{[\omega]} \alpha \phi Z + \alpha V_1 (C^*) - \alpha V_0 \phi + C_R \text{ where } C^* = C^*(\omega).
\] (5)

The necessary conditions for an interior maximum are given by the first- and second-order conditions, respectively:
\[
\phi Z + \alpha \phi_\omega Z + V_1 (C^*) - V_0 + \alpha V_1 C^* C^*_x + \phi_\omega C_R = 0,
\] (6)
\[
2 \phi_\omega Z + \alpha \phi_{\omega\omega} Z + 2 V_1 C^* C^*_x + \alpha V_1 C^* C^*_x^2 + \alpha V_1 C^* C^*_x^2 + \alpha C^* C^*_x C^*_x < 0.
\] (7)

Eqs. (6) and (7) allow us to differentiate our problem from the one studied in previous work (e.g., Shleifer and Vishny, 1986; Hirshleifer and Titman, 1990; Chowdhry and Jegadeesh, 1994). Generally speaking, previous models assume that $V_1(C^*) = \text{Constant} = V_0$. Thus, Eq. (6) changes to $\phi Z + \alpha \phi_\omega Z + \phi_\omega C_R > 0$, which
implies that \( x^* = \bar{x} \). In contrast, we include the additional negative term \( \alpha V_1 C_x C_a \) so that if the marginal impact of \( x \) on \( V_1 \) is sufficiently high we can have an equilibrium toehold that is lower than \( \bar{x} \). Also, note that this result holds regardless of the sign of the difference \( V_1(C_x) - V_0 \), as it is driven by the fact that \( V_1 \) is not a constant. Thus, it is the term \( \alpha V_1 C_x C_a \) that potentially leads to an equilibrium with both an interior solution to the optimal toehold and with positive profits from taking over. Put differently, due to the above difference, in our model the raider needs to take into account how the value of the firm following a failed takeover will be affected by the size of the toehold.

4. A specific takeover model and empirical implications

The purpose of this section is to derive more detailed empirical implications from our model. These implications will be discussed below. To this end we will make specific assumptions about (1) the distribution of the private benefits of control \( C \); (2) the functional form of the cash flows derived from the incumbent manager’s project; and (3) the probability of takeover success.

Assumption 3. At \( t = 1 \), the market believes that \( C \) is uniformly distributed between zero and \( C_M \).

Assumption 4. The cash flow at \( t = 3 \) generated by the project of a manager with private benefits \( C \) is given by \( \tilde{V}(C) = L((C_M - C)/C_M) \).

Assumption 4 captures the idea that higher private benefits imply lower future cash flows to target shareholders.

Assumption 5. Takeover success depends on whether the target manager supports the takeover and whether enough shareholders tender their shares. First, if the manager supports the takeover she derives a utility of \( \beta(V_1 + X) \), while if she opposes it then the takeover fails and her utility is \( \beta V_1 + C \). Second, the probability that enough shareholders tender their shares is (linearly) increasing in \( \alpha \). Specifically, we assume that probability of takeover success is

\[
\phi = \max \left\{ 0, \frac{k\alpha}{C_M} + \text{Prob}[\beta(X + V_1) > \beta V_1 + C] \right\} = \frac{C^*}{C_M},
\]

where \( C^* = \beta X + k\alpha \), and where we implicitly assume that \( C^* < C_M \). In Assumption 5 the main tradeoff for the target manager is between receiving the value \( V_1 + X \) on her equity stake if the takeover succeeds and receiving her private benefits of control if she decides to oppose the takeover. In addition, we assume that a larger toehold implies a higher probability of takeover success. While we do not explicitly model and derive the reason for this we rely on the result from past models. For example, Hirshleifer and Titman (1990) show that given the fraction of shares that the raider must acquire to obtain control (\( \omega \)) and that minority shareholders randomize between tendering their shares and not tendering, the more shares the
raider owns prior to the tender offer (higher \( a \)) the higher is the probability that he will obtain the rest of the shares (\( \omega - a \)) and successfully complete the takeover. Again, for our analysis the importance of this assumption is that it results in a takeover probability (ex ante) that is increasing in \( a \), consistent with many empirical papers.

When the takeover fails, the market and target shareholders update their beliefs about the distribution of the private benefits of control. In particular, shareholders rationally interpret the information content of a failed takeover in the following way: assuming for the moment that \( V_1 + X \geq V_1 + Z \) (i.e., that the raider does not underbid for the target), shareholders conclude that the takeover failed because \( C > C^* \). This affects their valuation of \( V_1 \).

**Lemma 1.** The post-failed-takeover target price, \( V_1 \), is given by

\[
V_1 = E \left[ \tilde{V}(C) \mid C^* < C < C_M \right] = \frac{L C_M - C^*}{2 C_M}, \quad \text{where} \ C^* = \beta X + k\alpha.
\]

Finally, we make a simplifying assumption regarding the initial value of the firm, \( V_0 \).

**Assumption 6.** In computing target firm value at \( t = 0 \) shareholders only consider the expectation of future cash flows under the incumbent.

Assumption 6 avoids the added complication of assessing the distribution of \( Z \) and its correlation with \( C \). Using this assumption we have, \( V_0 = E(\tilde{V}_1) = L/2 \).

### 4.1. Equilibrium

The raider solves,

\[
\begin{align*}
\text{Max}_{\{a, X\}} & \quad \phi \omega (Z - X) + a \phi X + \alpha (V_1 - V_0) + \phi C_R \\
\text{st.} & \quad \phi = \frac{C^*}{C_M}, \\
& \quad V_1 = \frac{L C_M - C^*}{2 C_M}, \\
& \quad C^* = \beta X + k\alpha, \\
\end{align*}
\]

and \( V_0 = \frac{L}{2} \).

First, we show that under an intuitive condition the optimal bid is to offer \( X^* = Z \).

**Lemma 2.** Assuming that \( C_R \) is sufficiently small, then if \( \bar{a} < \omega/2 \) the raider’s optimal bid is \( V_1 + X^* = V_1 + Z \).

**Proof.** See Appendix A.

Similar to Hirshleifer and Titman (1990) we analyze a setting in which the bid price is equal to the raider’s valuation. While in their work this is a result of focusing
on a separating equilibrium, in our model it is the result of the assumption that Z becomes public information at the time of the takeover contest.\footnote{Different bidding strategies will arise with asymmetric information over the value improvement. Models of bidding with asymmetric information include Giammarino and Heinkel (1986), Fishman (1988), Hirshleifer and Png (1989), Burkart (1995), Singh (1998), and Bulow et al. (1999).}

Since the post-takeover value $V_1$ is a linearly decreasing function of $z$, the raider’s profit function above is a second-degree polynomial. Thus, the existence of an interior maximum will depend on the sign of the coefficient of $z^2$. Proposition 1 characterizes the optimal toehold.

**Proposition 1.** The optimal toehold is given by

$$x^* = \begin{cases} 
0 & \text{for } 0 \leq \frac{L}{2} \text{ and } C_R \leq A_0 \\
\frac{C_R}{L - 2Z} - \frac{\beta Z}{2k} & \text{for } 0 \leq \frac{L}{2} \text{ and } A_0 < C_R \leq A_0 + A_1 \\
z & \text{for } 0 \leq \frac{L}{2} \text{ and } A_0 + A_1 < C_R \\
\bar{z} & \text{for } Z \geq \frac{L}{2}
\end{cases},$$

where

$$A_0 = \frac{L - 2Z}{2k} \beta Z > 0,$$

and

$$A_1 = \frac{L - 2Z}{2k} \bar{z} > 0.$$

**Proof.** See Appendix A.

Proposition 1 captures the main insight of the paper. In contrast to previous work in which $x^*$ is always equal to $\bar{z}$, we show that an interior maximum can exist. This result is achieved despite the fact that $Z$ is public information and that we allow the raider to purchase the toehold anonymously at the prevailing market price, $V_0$. The intuition for the result is based on the inherent difference between acquiring shares before the takeover and during the tendering stage. Unlike buying the shares during the tender offer stage, which is conditional on success, the raider owns the toehold even if the takeover fails (recall that we model a conditional tender offer, in which the raider acquires the shares tendered only if he obtains the majority needed for control). Since the combination of a failed takeover and a large toehold implies that the manager’s entrenchment level must have been high, a large toehold significantly reduces the post-failed-takeover target price. Since this is the price at which the raider sells his toehold he may find it optimal to limit the toehold size. Put differently, a failed takeover implies that the minimum level of managerial entrenchment, $C^*$, is higher than previously assessed by the market (greater than
zero). But a higher level of entrenchment also implies that the expected future value of the firm is lower. Thus, the raider might wish to buy less than \( \bar{z} \) if the marginal impact of buying more shares has a sufficiently negative effect on the post-failed-takeover price. Finally, note that although in our model all agents are risk neutral, having a risk averse raider would further reduce his incentive to acquire a toehold because the takeover success is uncertain.

Proposition 1 indicates that \( z^* \) can be less than \( \bar{z} \). However, it also shows that it can be optimal to have \( z^* = 0 \). This is because in our model the raider profits from both the toehold as well as from his private benefits of control (recall \( C_R > 0 \)). In practice, some raiders choose a zero toehold (e.g., Betton and Eckbo, 2000). Our model is consistent with this scenario as long as the raider’s takeover profits are not limited to his toehold (i.e., there are also private benefits).

4.2. Empirical implications for the optimal toehold

Having derived the optimal toehold, we now turn to analyzing the cross-sectional implications for how the optimal toehold relates to different characteristics of the economic environment. For the sake of interest we will focus on the parameter space in which \( 0 < z^* < \bar{z} \).

**Corollary 1.** *The optimal toehold, \( z^* \), decreases with the level of the target manager’s insider ownership, \( \beta \).*

Target managerial ownership and block ownership by an outside raider are negatively correlated. This is because a higher level of target manager ownership aligns the incentives of the manager and of minority shareholders and hence increases the ex ante likelihood that the takeover will succeed.\(^{10}\) While this increases the likelihood of a gain to the raider, it also implies that the market’s assessment of target value following a failed takeover will be lower (see Lemma 1). This lower firm value is the (marginal) cost of buying the toehold because it is the value at which the raider sells his toehold after the takeover fails. Thus, a higher \( \beta \) increases the concave profit but also the convex cost of buying a toehold. Hence, the net result is that a target with higher insider ownership will induce the raider to buy a smaller toehold. For empirical testing of this implication an obvious proxy for \( \beta \) would be the equity ownership stake of the target’s CEO.

**Corollary 2.** *The optimal toehold, \( z^* \), increases with the marginal impact of the toehold on the probability of takeover success, \( k \).*

The parameter \( k \) represents the marginal impact of the toehold on the ex ante likelihood of takeover success. When a toehold has a larger impact on takeover success, the raider’s incentive to buy a larger toehold increases. Mikkelson and

\(^{10}\)Recall that we assume that the manager’s ownership (of the target) is in the range where an increase in ownership increases the alignment of her interests with those of minority shareholders rather than her level of entrenchment. More generally, in the range of ownership in which higher target ownership increases entrenchment our comparative static results will have the opposite sign to that described in Corollary 1.
Partch (1989) find that bidders are more likely to acquire toeholds when the target firm is smaller. If a toehold has a larger impact on takeover success for smaller target firms, then their results are consistent with our prediction. However, liquidity considerations and the number of different shareholders in small versus large firms can also play a role and drive the result to the opposite direction. We do not have a clear view regarding how these will impact the parameter, $k$. Note that $k$ can also represent the power and influence of the raider on existing target shareholders. If so, proxies for the raider’s bargaining power can include the number of years the raider (or the raider’s financial advisors) has been engaging in such activity, his success rate in previous acquisitions, and his past performance. Namely, a more successful raider is likely to have more bargaining power.11

**Corollary 3.** The optimal toehold, $x^*$, increases with the raider’s private benefits of control, $C_R$.

As the raider’s private benefits of control increase it is more beneficial for him to take over the firm and hence he will increase his toehold to increase the likelihood of takeover success. This result is straightforward and is probably not unique to our specific model of takeovers. Possible proxies for the raider’s private benefits of control are the size of the target firm, and the position subsequently assumed by the raider in the target (i.e., whether he becomes the CEO, chairman of the board, or both).

### 4.3. Empirical implications for firm value following a failed takeover

In this subsection we discuss our model’s empirical implications for how firm value following a failed takeover should vary cross-sectionally. As above, these predictions are unique to our model and have not been tested before.

**Corollary 4.** The value of the target firm following an unsuccessful takeover, $V_1$, decreases with the bidder’s toehold, $x$. When controlling for the toehold, firm value (a) decreases with the target manager’s ownership, $\beta$, (b) decreases with the marginal impact of the toehold on takeover success, $k$, (c) increases with the expected severity of the agency problem in the target, $C_M$, and (d) decreases with the spread parameter, $Z$. Finally, when the toehold is endogenous, firm value decreases with the raider’s private benefits, $C_R$.

Corollary 4 is based on Lemma 1. The intuition for these results comes from the fact that any change in an exogenous variable that results in an ex ante increase in the probability of takeover success will also result in an ex post decrease in the value of the firm following a failed takeover. For example, the intuition for the result on the positive relation between $C_M$ and $V_1$ is that with a higher $C_M$ (or a higher expected level of target entrenchment) the ex ante probability of a successful

---

11This argument has been made in a different context by Hermalin and Weisbach (1998) who show that successful managers have more power over the board relative to managers who have not been successful in the past and hence lack bargaining power.
takeover is lower. This implies that a failed takeover has a smaller negative signal about the incumbent’s entrenchment level (relative to the expected level) and thus the price fall is smaller.

To test these predictions one can perform a cross-sectional regression on target prices following failed tender offers, and treat each of the exogenous model parameters in the corollary, including the bidder’s toehold, as an explanatory variable in the regression. Proxies for the magnitude of the agency problem and parameter $C_M$ can include the level of the target’s cash holdings, the amount of debt in its capital structure, and other corporate governance measures such as board size and number of board insiders that point to weak governance. The variable $Z$ can be computed by subtracting firm value following a failed takeover from the initial offer price. Proxies for the other variables were described earlier.

Empirical evidence suggests that following a failed takeover, the target’s price falls from pre-tender highs (e.g., Choi, 1991; Denis and Serrano, 1996; Safieddine and Titman, 1999). The results in Denis and Serrano (1996) suggest that firm value decreases due to (perhaps the update by shareholders of) managerial entrenchment. In addition, Safieddine and Titman (1999) find evidence that firm performance, following a failed takeover, decreases more for firms with larger insider holdings, which is consistent with Corollary 4. It is important to note, however, that while a decrease in firm value after a failed bid is consistent with our model it is not a necessary condition for our model to hold. Rather, the important feature for our model is that the marginal impact of the toehold on $V_1$ is negative.

4.4. Empirical implications for price changes and takeover premium

For any takeover contest one can define three event windows over which premiums (or price changes) are measured: (1) from a pre-announcement date (say a month before the bid is made) to the time of a public bid; (2) from a pre-announcement date to the time the takeover is (successfully) completed or terminated; and (3) from the time of the announcement of the bid to the time the takeover is (successfully) completed or terminated. Below we describe how these price changes should vary cross-sectionally.

**Corollary 5.** The price change from the pre-announcement date to the public announcement of the bid is given by $V_1 + \phi Z - V_0$. We find that this price change is decreasing in the size of the toehold. When allowing for an endogenous toehold we find that the price change is decreasing in insider ownership ($\beta$), the marginal impact of the toehold on takeover success ($k$), and in the raider’s expected private benefits of control ($C_R$), but is increasing in the severity of the agency problem in the target ($C_M$).

The intuition for these results comes directly from the assumptions on how the endogenous variables affect the takeover probability and on how these variables (e.g., the raider’s private benefits of control) affect the optimal toehold. Note that since $V_1 < V_0 < V_1 + Z$, then depending on how the initial price, $V_0$, is defined one could have a negative price runup. While this is an empirically unsatisfying implication, it can be easily fixed if, for example, we assume that the takeover also
conveys other positive information (such as the existence of another raider) so that \( V_1 > V_0 \) even though the takeover fails. The important part for our analysis is not that \( V_1 < V_0 \) but that \( V_1 \) decreases in \( z \).

**Corollary 6.** The price change from the pre-announcement date to the successful completion of the takeover is given by \( V_1 + Z - V_0 \). The price change is decreasing in the size of the toehold and, conditioning on the size of the toehold, in insider ownership (\( \beta \)). When allowing for an endogenous toehold we find that the price change is decreasing in the marginal impact of a toehold on takeover success (\( k \)) and in the raider's private benefits of control (\( C_R \)) but increasing with the severity of the agency problem (\( C_M \)).

The main difference between Corollaries 5 and 6 is that in Corollary 6 the realized price does not incorporate shareholders’ expectations of the likelihood of takeover failure. The general intuition for these results is that the price being offered to shareholders is an increasing function of their valuation of the firm under the incumbent manager (conditional on takeover failure). Thus, the lower is this expected value (\( V_1 \)) the lower is the premium shareholders will receive in a successful takeover. Note that the decrease of the premium with the toehold is also predicted by the models of Shleifer and Vishny (1986) and Hirshleifer and Titman (1990) and is consistent with the empirical evidence in Betton and Eckbo (2000).

**Corollary 7.** The price change from the announcement of the bid to the successful completion of the takeover is given by \( V_1 + Z - (V_1 + \phi Z) \). This price change is decreasing in the size of the toehold and, conditioning on the size of the toehold, decreasing in insider ownership (\( \beta \)). When allowing for an endogenous toehold we find that the price change is decreasing in the marginal impact of a toehold on the success probability (\( k \)) and in the raider’s private benefits of control (\( C_R \)) but increasing with the severity of the agency problem (\( C_M \)).

For this event window the price change captures the shareholders’ assessment of the probability that the takeover will fail and that they will lose the spread value, \( Z \). The intuition for this result comes trivially from the relation between model parameters and the probability of takeover success.

5. Model extensions

In this section we extend the model in two ways. First we extend the model to the case in which the raider can profit both through acquiring a toehold anonymously and through his ability to dilute minority shareholders (while assuming that \( C_R = 0 \)). The purpose of this extension is to demonstrate that our main results regarding optimal toeholds are robust to alternative raider profit and payoff structures. In particular, we demonstrate an additional way in which our model can generate takeover attempts where the optimal toehold is zero. One of the major puzzles facing existing takeover theories is the empirical finding that the median size of toeholds is
zero (e.g., Bradley et al., 1988; Betton and Eckbo, 2000). Thus, one of the points of our model is to offer an explanation for the existence of a zero toehold.

In a second extension of the basic model we allow for multiple bidders (two) and analyze the impact of the additional bidder on the initial toehold. In this extension we identify an additional benefit of acquiring a toehold (or becoming a block shareholder of the target) in an environment where there are other potential bidders.

5.1. Dilution of minority shareholders

In this section we follow the model in the previous section and add an additional assumption about the raider’s ability to dilute minority shareholders.12

**Assumption 7.** If the raider takes control of the firm he can dilute firm value by an amount $\delta Z$.

Thus, when $\delta = 1$ the raider can completely dilute minority shareholders (effectively expropriating all of the value spread) and when $\delta = 0$ we are back to the previous model. As analyzed in Grossman and Hart (1980) the ability to dilute minority shareholders allows the raider to bid below the full post takeover value and still make it worthwhile for shareholders to tender. With dilution, the raider does not need to bid the full premium, $V_1 + Z$, to overcome the free-rider problem. Rather, as long as he offers a price $V_1 + X \geq V_1 + (1 - \delta)Z$, minority shareholders will tender.

The raider’s expected profit is given by

$$
\Pi = \phi[\delta(1 - \omega)Z + \omega(Z - X) + \alpha X] + \alpha(V_1 - V_0). \tag{9}
$$

As before we wish to focus on the case in which the raider does not “overbid”, i.e., offers $X > (1 - \delta)Z$. Lemma 3 provides the condition for this.

**Lemma 3.** If $\beta < (\omega - \delta)/(1 - \delta))/2$ then the raider’s optimal bid is equal to $X^* = (1 - \delta)Z$.

**Proof.** See Appendix A.

**Proposition 2.** Under the condition specified in Lemma 3, and for $Z(1 - \delta) < L/2$, the optimal toehold is given by

$$
\alpha^* = \frac{\delta Z}{2(L/2 - (1 - \delta)Z)} - \frac{\beta(1 - \delta)Z}{2k}.
$$

---

12One example of dilution is two-tiered offers (e.g., Comment and Jarrell, 1987), in which the front-end tender price is higher than the back-end, cleanup price (this can be the merger offer price). The problem with this practice is that shareholders who do not tender in the first stage can ask a court to enforce the “fair price rule” so that the back-end price is at least as high as the front-end price (e.g., Jarrell and Poulsen, 1987; Ravid and Spiegel, 1999).
Proof. See Appendix A.

Proposition 2 is correct as long as the parameters are such that \(0 \leq \alpha \leq \bar{\alpha}\). With dilution, there are opposing effects on the optimal toehold. On the one hand, dilution reduces the bid price, and hence the target price following a failed takeover does not drop as much as before. This implies that a larger toehold is more profitable. On the other hand, dilution reduces the positive profit on the toehold when the takeover succeeds because the dilution threat applies to all shareholders, including the target shares owned by the raider himself. Thus, when buying a larger toehold the raider is diluting himself. This tends to lower the optimal toehold. Which of these two effects dominates will determine whether the optimal toehold is increasing or decreasing with the threat of dilution.

To emphasize again, the main additional result derived in this subsection, and which appears in Proposition 2 and Eq. (9), is that with dilution we can observe takeovers that are profitable to the raider and in which the raider optimally chooses to buy a zero toehold. This, along with Proposition 1 (in which \(C_R > 0\)), can help explain why in empirical studies the median size of the toehold is indeed zero.

5.2. Takeover model with a rival bidder

In this section we relax the assumption that there is only one bidder in the takeover process and instead we assume that a rival bidder, \(R\), may also enter the control contest. Our focus, however, will remain on the initial bidder, \(B_1\), and on how the potential presence of a rival bidder affects his toehold strategy. All other assumptions made earlier carry over to this section. Other takeover models with multiple bidders (e.g., Burkart, 1995; Singh, 1998; and Bulow et al., 1999) show a positive strategic effect of a toehold, but they assume bidders exogenously start with toeholds of different sizes rather than solving for the optimal toehold.

For simplicity, we make the following assumption.

Assumption 8. With probability \(P \in (0, 1]\), the rival bidder has a value improvement \(Z_H\) and with probability \(1 - P\) his value improvement is \(Z_L\). Furthermore,

\[Z_H + V^R_1(Z_H) > Z + V_1 > Z_L + V^R_1(Z_L),\]

where \(V^R_1(\bullet)\) is the value of the firm if the rival bidder makes the tender offer and the offer fails.

The above implies that \(R\) will enter the contest and make a tender offer if and only if he has the high valuation, \(Z_H\). In that case \(R\) becomes the only bidder facing the target manager and the game continues as before. Fig. 2 describes the new sequence of events.

To simplify things further we assume that \(R\)’s toehold, \(z_R\), is determined exogenously. One can regard \(R\) as an existing blockholder of the target (e.g., Shleifer and Vishny, 1986) who becomes interested in taking over the firm himself once he sees a tender offer filed by \(B_1\).
Solving for the equilibrium toehold of B1 is similar to that of the single bidder’s problem specified in Proposition 1, and we leave the details to Appendix B. We find that the main difference is that adding the rival bidder changes the initial raider’s profit function since the value of the firm will incorporate the possible entry of R and the outcome of the subsequent contest and success (or failure) of the takeover. In particular, when the initial bidder observes that the rival bidder enters the takeover contest, he knows that the rival bidder’s value improvement is higher than his own and that selling his toehold to the rival bidder yields higher profits as compared to taking over himself. Betton and Eckbo (2000) find that the expected payoff to initial bidders is significantly positive when a rival bidder wins the auction and control contest, consistent with our model.

More specifically, we find that the interior solution of B1’s toehold is larger relative to the case without a rival bidder and that it increases when the likelihood of a rival bidder entering the contest increases (parameter \( P \)), or when the rival bidder’s value improvement increases (\( ZH \) increases). These results are intuitive as an increase in either of the two parameters increases B1’s payoff from selling the toehold to the rival bidder. One implication of this result is that initial bidders will have a larger toehold when the rival bidders win the auction with a higher bid. To proxy for the probability of a high-value rival bidder entering the contest, one can use the frequency of other tender offers (say within that industry) that attract competition.

6. Conclusion

Empirical evidence suggests that the majority of raiders do not acquire a large toehold prior to initiating a tender offer. This fact stands in sharp contrast to most existing theoretical papers which argue that it is always optimal for the raider to acquire the maximum toehold that is allowed by regulation prior to being forced to report his intentions (two notable exceptions, mentioned earlier, are Chowdhry and Jegadeesh (1994) and Ravid and Spiegel (1999)). We solve for the interior optimal size of the toehold by demonstrating a potential cost to buying a toehold. This cost relies on the endogenous determination of the market value of the firm following a failed takeover. In particular, we show that buying a larger toehold reduces the value...
of the firm following a failed takeover whenever takeovers fail due to managerial entrenchment. This happens precisely because the probability that a takeover will succeed is increasing in the size of the toehold. When raiders have private benefits of control or when they are able to dilute minority shareholders we further show that the optimal toehold can actually be zero.

We analyze the cross-sectional variations in the size of the toehold and show how toehold size varies with managerial ownership stake and with measures of the agency cost. Finally, we derive cross-sectional predictions on the value of the firm following a failed takeover, as well as on the bid premium and the magnitude of target price changes for different event windows surrounding the takeover contest.

Appendix A

A.1. Proof of Lemma 2. Since we know that \( X \geq Z \) we need to show that \( X^* \) is not larger than \( Z \). A sufficient condition for this is that the partial derivative of the raider’s profit function \( \Pi \) with respect to \( X \) is negative at the point \( X = Z \). But

\[
\frac{\partial \Pi}{\partial X} = \frac{\partial}{\partial X} [\phi \omega (Z - X) + \alpha \phi X + \alpha (V_1 - V_0) + \phi C_R].
\]

Replacing in the functions for \( \phi \) and \( V_1 \) we have,

\[
\text{Sign} \left[ \frac{\partial \Pi}{\partial X} \right]_{X = Z} = \frac{\beta}{C_M} (\alpha Z + C_R) - \frac{\beta Z + k \alpha}{C_M} (\omega - \alpha) - \alpha \frac{\beta L}{2C_M}
\]

\[
= 2\beta Z (2\alpha - \omega) - \alpha [2k(\omega - \alpha) + \beta L] + 2\beta C_R.
\]

As long as \( C_R \ll 1 \), a sufficient condition for the above expression to be negative is \( \tilde{\alpha} \ll \omega / 2 \). \( \Box \)

A.2. Proof of Proposition 1 (optimal toehold in single bidder case). The solution to the first-order condition (FOC) is a maximum only if the second-order condition (SOC) is negative. The SOC is equal to \((Z - L/2)k/C_M\). Therefore, for \( Z \geq L/2 \) the objective function is maximized at \( \tilde{\alpha} \). When \( Z < L/2 \) the second-order condition is negative and the solution to the first-order condition gives the optimal toehold (as long as the optimum is between zero and \( \tilde{\alpha} \)).

Taking the FOC of the profit function, we have \( \phi Z + V_1 - V_0 + (\alpha Z + C_R) d\phi/dz + \alpha (dV_1/dz) = 0 \).

Replacing the expression for \( V_1 \) and for \( \phi \) yields \( \alpha^* = C_R / (L - 2Z) - \beta Z / 2k \). The remaining conditions on \( C_R \) divide the parameter space into regions where this expression is less than zero, between zero and \( \tilde{\alpha} \), and greater than \( \tilde{\alpha} \). \( \Box \)

A.3. Proof of Lemma 3 and Proposition 2 (optimal toehold with dilution). We first show that \( X = (1 - \delta)Z \) in equilibrium. Similar to the proof of Lemma 1, we
differentiate the expected profit w.r.t. $X$:

$$
\frac{\partial \phi}{\partial X} [\partial (1 - \omega)Z + \omega (Z - X) + \alpha (V_1 + X)]
+ \phi \left[ -\omega + \alpha \left( \frac{\partial V_1}{\partial X} + 1 \right) - \frac{\partial \phi}{\partial X} \alpha V_1 + (1 - \phi) \frac{\partial V_1}{\partial X} \right].
$$

Replacing the expression for $V_1$, $\phi$, $\partial \phi / \partial X$, and $\partial V_1 / \partial X$, and letting $X = (1 - \delta)Z$, we have

$$
\left. \frac{\partial \Pi}{\partial X} \right|_{X = (1 - \delta)Z} = 2\beta Z [\delta + 2(1 - \delta)\alpha - (1 - \delta)\omega] - \alpha [2k(\omega - \alpha) + \beta L].
$$

A sufficient condition for the above expression to be negative is $\delta + 2(1 - \delta)\alpha - (1 - \delta)\omega < 0$, which is implied by the condition in Lemma 3.

Next, to find the optimal toehold, we take the first-order condition of the profit function w.r.t. $\alpha$, $(d\phi / dz)(\delta Z + \alpha (1 - \delta)Z) + \phi (1 - \delta)Z + V_1 - V_0 + \alpha (dV_1 / dz) = 0$.

Replacing the expression for $d\phi / dz$ and $dV_1 / dz$, we get an equation similar to what we obtained in Proposition 1 (optimal toehold without dilution):

$$
\alpha \left( \frac{2k(1 - \delta)Z}{C_M} - \frac{kL}{C_M} \right) = V_0 - \frac{k\delta Z + \beta (1 - \delta)Z^2}{C_M} + \frac{(1 - \delta)\beta LZ}{2C_M} - \frac{L}{2}.
$$

Simplifying and rearranging, we obtain the expression in Proposition 2.  

\textbf{Appendix B}

\textbf{B.1. Optimal toehold with multiple bidders}

Following the model set up in Section 5.2, in particular Assumption 8, the initial bidder’s (B1) optimization problem, assuming $C_R = 0$, is

$$
\text{Max}_{\phi} \quad (1 - P) [\alpha \phi Z + \alpha V_1] + P [\alpha (\phi^R Z_H + V_1^R(Z_H))] - \alpha V_0,
$$

s.t. $\phi = \frac{k\alpha + \beta Z}{C_M}$, $C^* = \beta Z + k\alpha$, $V_1 = \frac{L C_M - C^*}{2C_M}$ and $V_0 = \frac{L}{2}$.

\textbf{Proposition 3.} With a rival bidder, for $Z < L/2$, the optimal toehold of the initial bidder is

$$
\alpha^* = \frac{P}{1 - P} \frac{C_M - \frac{D}{2k}}{L - 2\frac{\beta Z}{2k}},
$$

where $D = 2(\phi^R Z_H + V_1^R(Z_H) - V_0) > 0$. Moreover, $\alpha^*$ increases when $P$ increases or when $D$ increases.
Proof. A simple first-order condition similar to the proof of Proposition 1 above will yield the expression for the optimal toehold. The two comparative statics are obvious from the expression. Furthermore, if we use the expressions $\phi^R = (\beta Z_H + k x^R)/C_M$ and $V_1^R = (L/2C_M)(C_M - (\beta Z_H + k x^R))/C_M$, we can show that $x^*$ increases with $Z_H$. Finally, comparing the expression here for $x^*$ with that from Proposition 1, we can show that when $(P/(1 - P))C_MD/2k > C_R$, which can be satisfied if $P$ and $Z_H$ are large enough and $C_R$ not too large relative to $C_M$, then the optimal toehold is higher with the presence of a rival bidder than without.

References


